Stochastic Programming Model for Supply Chain Design with Short Response Time

Thanatorn Sinphatisirikul
thanatorn.sin@gmail.com

Aussadavut Dumrongsiri
aussadavut@gmail.com
Sirindhorn International Institute of Technology (SIIT), Thammasat University, Thailand

Abstract

This research studies the impact of demanduncertainty to the supply chain design by using stochastic programming approach. Each potential location has two modes of supply: long response time used before the demand is realized and short response lead time mode (with higher cost) used after the demand is realized. The capacity of each model will be optimally determined from the model. This means that at each location allow the manufacturer to install machines to produce in a large quantity at low cost (due to economy of scale) and keep in the internal warehouse or to install flexible rapid response machines to produce with short lead time at high cost after the demand shortage is expected. Moreover, the model allows different production cost functions which the unit cost could be different when production quantity is different using piecewise function. A stochastic programming model is developed to handle the situation explained. We have conducted 3 experiments to test the model in different perspective; (1) Sampling test – to test the effect of the sample size to the result, (2) Parameter variation – to test how each variable affect the results, (3) Cost function testing – to reveal the property of different cost function.

Introduction

Every business relies on one of the most important factor which is the demand of product. Manager has to delicately plan and adjust the supply chain that involved by the company in order to satisfy the demand, minimize the cost and maximize the profit. However, in practical, demand is very difficult to control and sensitive to other environmental factors. Uncertainty of demand has crucial impact not only for the single company, but for the whole stream of supply chain. The mismatches between demand and supply can be very destructive. A proposed research has proven that the confliction between supply and demand result in 30% reduction of company's income (Hendricks et al., 2005). Bullwhip effect is also one of the major problems obviously affected by the demand uncertainty. Bullwhip effect is the distortion of demand upstream and creates the massive deviation of order along the supply chain stream (Hau L. lee, 1997).

According to the problem, *short lead time (LT)* response of production is highlighted as one of the methods forhandling the uncertainty of demand. Short LT response has pros and cons. It helps catalyzing the response rate of demand in term of time and quantity, but in the other hand, it costs more on many directions to afford the specialized technology.

Hence, this research diagnoses and manipulates the uncertainty of demand by developing stochastic programming model. The model is two echelons supply chain with single-commodity, multi-facilities and multi-retailers. In addition, we have implanted short LT response facilities in order to handle the demand fluctuation more effectively. Moreover, the model also focuses on different production cost function which the price is different when purchase in different amount by developing the piecewise function to handle various cost curve segmentation.

Literature review

We have reviewed some literatures on 3 main bases: (1) Facility location models with supply chain context (2) Stochastic programming (with demand uncertainty) and (3) Piecewise function. These 3 contents are the main contribution of our research.

First, for facility location paper review, Thanh et al. (2007) proposed a mixed integer linear programming of multi-level, multi-period, multi-commodity of production and distribution with deterministic demand. They also applied the model to handle the increasing of demand. Melo et al. (2004) presented dynamic multi-commodity capacitated facility location problem which figure the relocation of initial facilities that originally operated over the planning horizon. Moreover, the paper also concern about fluctuation of seasonal demand. For the paper of Jouzdani et al. (2012), fuzzy linear programming is developed which focuses on dynamic dairy facility location and supply chain planning of dairy products. In addition, non-linear mixed integer programming is implemented on traffic congestion. Hassanzadeh et al. (2012) considered mixed integer linear programming for closed-loop supply chain network. The model also regarded on environmental factors by implementing e-constraint and weighted sums methods.

Second, from reviewing on stochastic programming, Kaki et al. (2014) proposed stochastic problem applying newsvendor concept with distribution function, cumulative function and expected value. They focused on the affected of interdependent supply and demand uncertainty. They implemented scenarios based on copula functions which valid to handle linear and nonlinear dependences. Also, capacity reservation contract was used to compromise the uncertainty of demand and supply. Xu and Zhang (2013) introduced mixed-integer bi-level programming model and employs the iterative-optimization method. The bi-level programming divided into 2 parts: (1) the upper model is the logistics network design (LND). (2) The lower model is the order quantity determination (OQD). Chouinard et al. (2007) presented two-stage stochastic model for supply loops design. Linear regression and normal distribution are utilized to represent the uncertainty of demand. Furthermore, they extent heuristics on sample average approximation (SAA) featuring the Monte Carlo sampling methods. Next, Goh et al. (2007) proposed multi-stage stochastic convex programming applying probability for demand uncertainty. Moreover, they also used Moreua-Yoshida regularization and designed algorithm for interpreting of multi-stage global supply chain network problem. Lastly, for Wang et al. (2011), they developed two genetic algorithms, genetic algorithm with linear programming and genetic algorithm with greedy heuristics to examine bi-level stochastic programming.

Third, for the review of piecewise function and cost function, Da Lu (2010) proposed non-linear mixed integer programming on facility location problem with economies of scale and congestion and also proposes the Lagrangian solution approach. In addition, this paper interpret various type of cost function i.e. convex cost function, concave cost function and s-shaped cost function. Diabat and Theodorou (2014) presented mixed integer non-linear programming for single-warehouse multi-retailer inventory problem. They utilized piecewise function to convert non-linear to linear term. They also compare the result with other proposed Lagrangian relaxation results regardless any extraordinary algorithms. Chan et al. (2002) introduced inventory and transportation strategieson satisfying demand variation. Piecewise linear function is utilized for expressing quantity discounts function, incentive of volume based pricing and conceptual economies of scale.

Problem description

For this research, we have created a scenario of two echelons supply chain with 3 representative sites of facility serving *product A* to 20 dispersed customers. Each location site has potential to construct 2 types of facility (1 for each type) which are short LT response facility and long LT response facility. In fact, the short LT response facility produce the same exact product as the long LT, but it performs with shorter lead time and able to response more flexibly during critical time. Moreover, as it has more sophisticated technology and machines to perform the according specialty, it also cost more to build one. Both type of facility has its own production cost function represented in term of piecewise function with 3 segments, in which all of the 3 segments may or may not all be opened. In accordance to the piecewise function, the 2nd and the 3rd segment can be opened only if their previous segment is opened. Next, we divide the production schedule into 2 phases; (1) regardless demand uncertainty and (2) featuring with demand uncertainty. For the first period, only long LT response facility is opened for advanced production before receiving the real demand data, which will not take part on the fluctuation of future demand. This is scheduled as the stand by product. Then, for the second period, the fluctuation of demand will be considered and determine the production for short LT response facility in order to maximize sales and minimize costs, especially the shortage and overage amount. Ultimately, as the facilities are determined, the customer's demand will be served based on the decision made previously allocating which customer will be serving by which facility.

Mathematical Model (2 stages stochastic programming)

$Indexes:$
$i = demand\ node$
$j = facility\ node\ (JL: Long,\ JS: short)$
$k = segment\ k\ of\ production\ cost$
$s = scenario\ of\ second\ stage$
$Parameters:$
$c_{ij} = transportation\ cost\ from\ j\ to\ i$
$l_{jk} = production\ cost\ (slope)\ per\ unit\ using\ line\ segment\ k\ at\ location\ j$
$ac_{jk} = marginal\ fixed\ cost\ to\ construct\ faciliy\ of\ size\ k\ at\ location\ j$
$(additional\ cost\ in\ expand\ capacity\ from\ size\ k-1\ to\ size\ k)$
$fc_{jk} = fixed\ cost\ to\ construct\ faciliy\ at\ location\ j$
$p = probability\ of\ scenarios$
$Sh = Shortage\ cost\ per\ unit$
$Ov = Overage\ (holding)\ cost\ per\ unit$
$D_{is} = Demand\ of\ product\ at\ location\ i\ under\ scenario\ s$
$L_{jk} = Length\ of\ segment\ k\ of\ production\ cost\ at\ location\ j\ or$
$(Additional\ capacity\ if\ size\ k\ is\ selected\ over\ size\ k-1\ at\ location\ j)$
$UP = Unit\ selling\ price$

Decision Variables :

First stage :

x_{ij} = *quantity shipped from* j *to* i *using long lead time facility*

δ^{x}_{jk} = *quantity produced at location* j *using long lead time under segment* k *of production cost*

$w_{jk} = 1$ *if quantity produced is at upperbound* k (*full capacity of segment* k) *of production cost* ($\delta^{x}_{jk} = L_{jk}$ *or* $\delta^{y}_{jk} = L_{jk}$), 0 *otherwise*

$OPEN_j = 1$ *if a facility is open at location* j, 0 *otherwise*

Second stage :

y_{ijs} = *quantity shipped from* j *to* i *using short lead time facility under scenario* s

δ^{y}_{jks} = *quantity produced at location* j *using short lead time under segment* k *of production cost*

SQ_{is} = *shortage quantity at location* i *under enario* s

OQ_{is} = *overage quantity at location* i *under scenario* s

$$\text{Minimize} \qquad Up\left(\sum_{i}\sum_{j}^{JL} x_{ij} + \sum_{s} p \sum_{i}\sum_{j}^{JS} y_{ijs} - \sum_{s} p \sum_{i} OQ_{is}\right) -$$

$$\left(\begin{array}{l} \displaystyle\sum_{i}\sum_{j} c_{ij}x_{ij} + \sum_{j}^{JL}\sum_{k} l_{jk}\delta^{x}_{jk} + \sum_{j}\sum_{k} w_{jk}ac_{jk} + \sum_{j} OPEN_j fc_j + \\ \displaystyle\sum_{s} p_s\left(\sum_{i}\sum_{j} c_{ij}y_{ijs} + \sum_{j}^{JS}\sum_{k} l_{jk}\delta^{y}_{jks} + \sum_{i} SQ_{is}Sh + \sum_{i} OQ_{is}Ov\right) \end{array}\right)$$

Subject to

$$ct1: \sum_{j} x_{ij} + \sum_{j} y_{ijs} + SQ_{is} - OQ_{is} = D_{is}, \forall i, s$$

$$ct2: \sum_{i} x_{ij} = \sum_{k=1}^{K} \delta^{x}_{jk}, \forall j \in JL$$

$$ct3: \sum_{i} y_{ijs} = \sum_{k=1}^{K} \delta^{y}_{jks}, \forall j \in JS, s$$

$$ct4: \sum_{i} x_{ij} \le OPEN^{x}_{j} M, \forall j \in JL$$

$$ct5: \sum_{i} y_{ijs} \le OPEN^{y}_{j} M, \forall j \in JS, s$$

$$ct6: L_{j1} w_{j1} \le \delta^{x}_{j1} \le L_{j1}, \forall j \in JL$$

$$ct7: L_{k} w_{jk} \le \delta^{x}_{jk} \le L_{jk} w_{j,k-1}, \forall j \in JL, \forall k \in \{2,3,\ldots,K-1\}$$

$$ct8: 0 \le \delta^{x}_{jK} \le L_{jK} w_{j,K-1}, \forall j \in JL$$

$$ct9: L_{j1} w_{j1} \le \delta^{y}_{j1s} \le L_{j1}, \forall j \in JS, s$$

$$ct10: L_{k} w_{jk} \le \delta^{y}_{jks} \le L_{jk} w_{j,k-1}, \forall j \in JS, \forall k \in \{2,3,\ldots,K-1\}, s$$

$$ct11: 0 \le \delta^{y}_{jKs} \le L_{jK} w_{j,K-1}, \forall j \in JS, s$$

$$ct12: x_{ij}, y_{ij}, \delta^{x}_{jk}, \delta^{y}_{jks}, SQ_{is}, OQ_{is} \ge 0$$

$$ct13: w_{jk}, OPEN_{j} \in \{0,1\}$$

The objective function of the model is to maximize the profit from sales of product from short LT response and long LT response (excluding the overage amount) and deduct by various cost of long LT response and short LT response i.e. transportation cost (long LT), production cost (long LT), capacity additional cost, facility fixed cost, and with the probability distribution for transportation cost (short LT), production cost (short LT), shortage cost and overage cost. Constraint (1) represents the demand satisfaction. Constraint (2) determines total production and delivery from all of the long LT response facility. Constraint (3) determines total production and delivery from all of the short LT response facility. Constraint (4) locates the long LT response facility. Constraint (5) locates the short LT response facility. Constraint (6, 7, and 8) represents the piecewise function of long LT response facility’s production capacity. Constraint (9, 10, and 11) represents the piecewise function of short LT response facility’s production capacity. Constraint (12) determines the non-negativity variables. Lastly, Constraint (13) determines the binary variable.

Numerical experiment

Example: Base case

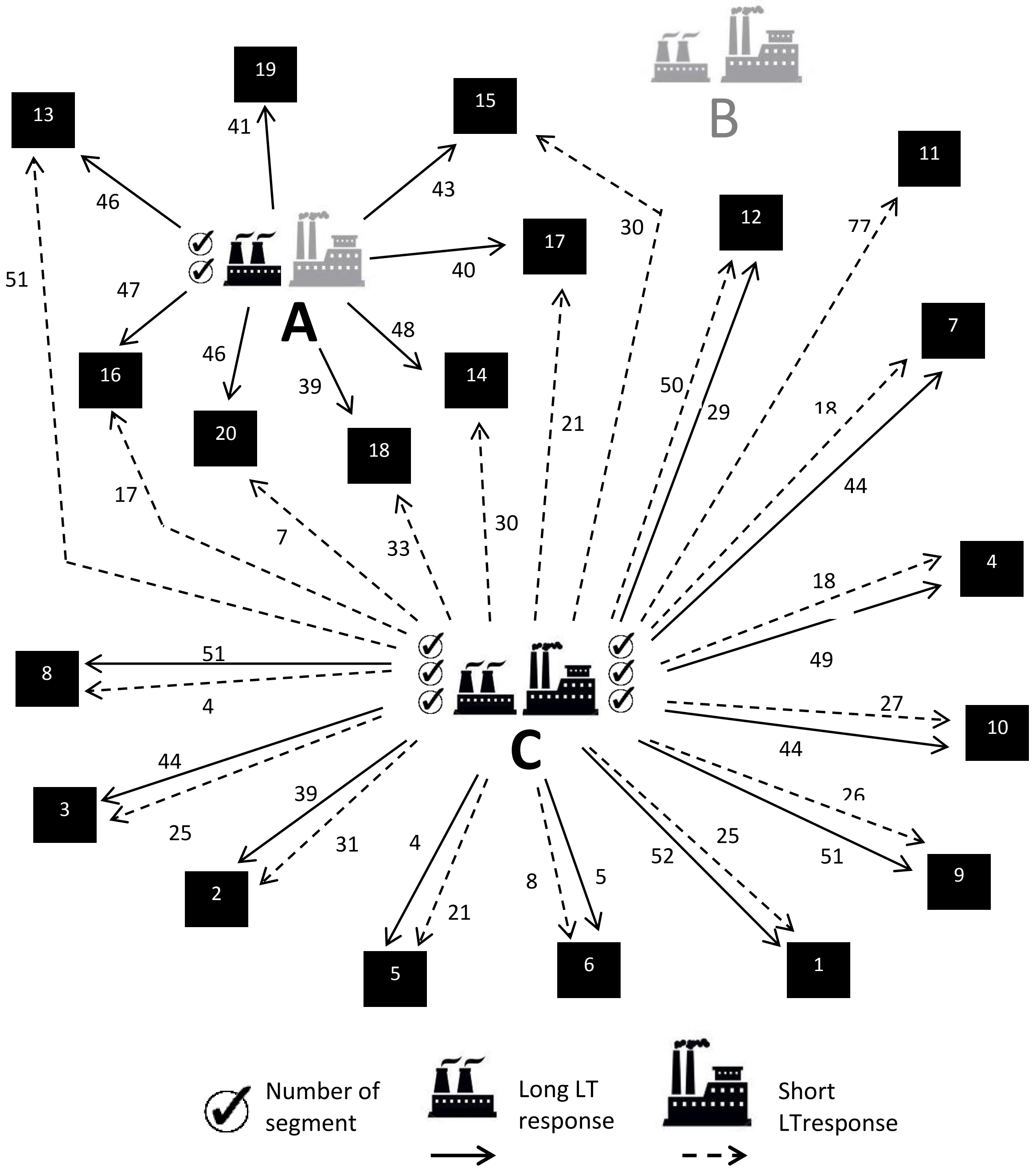


Figure 1: Base case scenario's result

Table 1:
Results of the base case

Type	*Location ⍰*									*Production amount*		*Avg. Shortage amount*	*Avg. Overage amount*	*Total Profit*
	A			*B*			*C*							
	Segmentation									*Individual*	*Total*			
	1	*2*	*3*	*1*	*2*	*3*	*1*	*2*	*3*					
Long	■	■	□	□	□	□	■	■	■	850	1339.07	50.94	3.72	10,604,193
Short	□	□	□	□	□	□	■	■	■	489.07				

Table 2:
Input data of the base case

Type	*Facility fixed cost*	*Capacity additional cost*	*Selling Price per unit*	*Production cost per unit (S-shaped)*		
				Segment 1	*Segment 2*	*Segment 3*
Long	5M	1M	38,000	15,000	8,000	18,667
Short	7M	1M				

Overage Cost per unit	*Shortage Cost per unit*	*Transportation cost per unit (Random)*		*Demand Distribution*	
		min	*max*	*Mean*	*SD*
6,000	0	45	140	70	10

Figure 1 and table 1 show the result of base case scenario which is utilized in most of the experiment afterward. It shows all the significant decisions and distribution mapping of the whole situation. Table 2 reveals all of the input data of the base case that applied to the model for solving.

For the experiment part, this research introduces 4 separated experiments: (1) Sampling test, (2) Parameter variation, (3) Cost function testing and (4) Pair-T test. For these experiments, data variation is mainly applied to conduct those experiments. This part visualizes how each of input data influentially affects certain situation. Moreover, some experiments are developed by constructing some statistical analysis to obtain concrete and reliable results.

We utilize IBM ILOG CPLEX 12.1 for programming and solving.

Sampling test

Theoretically, larger samples increases the chances of exposing specific mean difference because they are more reliably reflecting the population mean. This experiment is conducted to test how the size of sampling consequently contributed the results. We test to see that if the sample size is bigger, the deviation would be smaller. Distribution of customer's demand in each scenario considered to be the sample size of the problem. Every scenario is created based on the same distribution (mean and standard deviation).

Parameter variation

In this experiment, we vary various parameters of the model to acquire quantitative and qualitative difference of the results. For the quantitative difference, the comparison can obviously be shown by the numerical differences of major quantitative decisions i.e. total profit, amount of production, average shortage amount and average overage amount. Then, for the qualitative difference, we try to test how the physical decisions are changed according to the variation of parameter i.e. location of facilities and segmentation of facilities.

Cost function testing

In this part, we develop a test to analyze the changes of the quantitative and qualitative results when different distribution of production cost function is implemented. We simulate 4 different characteristics of cumulative cost function i.e. (1) Convex cost function (2) Concave cost function (3) S-shaped cost function and (4) Stable cost function. Each cost curve is divided into 3 segments which each segment is refined according to the type of certain function.

Results and Analysis

This part is devoted to reveal the result of according experiments. We analyze the results in term of both quantitative and qualitative resolution. As we mentioned, quantitative analysis is focusing on profit and production and qualitative is targeting on facility location and segmentation allocation.

1.1 Sampling test

The experiment uses data of the base case. Each number of scenarios is retrieved from average of 5 replications (rep.).

Table 3:
Sampling test to illustrate the change in deviation

Number of S	*Rep.*	*Profit*	*Avg. cost*	*SD*	*time*	
100	1	10,895,847.15	10,894,615.79	25,106.82	1.37	mins
	2	10,922,218.58			1.35	mins
	3	10,861,803.73			1.31	mins
	4	10,914,954.62			1.44	mins
	5	10,878,254.85			1.41	mins
200	1	10,857,082.45	10,879,163.00	21,308.97	7.08	mins
	2	10,903,527.42			5.36	mins
	3	10,870,304.00			6.02	mins
	4	10,864,631.82			7.13	mins
	5	10,900,269.31			8.33	mins
300	1	10,889,839.32	10,883,290.84	8,740.92	19.39	mins
	2	10,870,616.34			15.42	mins

	3	10,878,082.69			18.56	mins
	4	10,891,274.46			24.82	mins
	5	10,886,641.38			27.24	mins
400	1	10,886,558.30	10,883,616.65	3,573.61	52.23	mins
	2	10,884,037.59			50.00	mins
	3	10,887,485.63			55.14	mins
	4	10,879,247.36			57.55	mins
	5	10,880,754.38			53.22	mins

According to table 3, we focus on the change of standard deviation. We can obviously illustrate that as the size of the sample increase, standard deviation decline. The standard deviation is descending from 25,106.82 to 3,573.61 as the number of sample ascending from 100 to 400 respectively. For average profit, there is no relationship or trend among the scenarios, they are all rely on the random distribution of demand which is unnecessary to be concerned in this part. Hence, it is statistically consistent with the sampling theory, so we can support that bigger sample size could produce more refined and precise results. In the other hand, for bigger sample size, it reasonably requires much longer computational time.

Parameter variation

The experiment uses the base case with 200 scenarios. For some of the parameter, the results are reasonably deviate according to general relationship of the parameter which are direct and inverse relationship. Hence, we decide to capture the some parameter of this experiment that highlight the significance modification of the result.

Unit Price

Table 4:
Results of unit price variation

Unit Price	*Type*	*Location ⍰*									*Production amount*		*Avg. Shortage amount*	*Avg. Overage amount*	*Total Profit*
		A			*B*			*C*							
		Segmentation									*Individual*	*Total*			
		1	*2*	*3*	*1*	*2*	*3*	*1*	*2*	*3*					
30,000	Long	■	■	■	□	□	□	■	■	■	1000	1000	399	12.71	1,454,690
	Short	□	□	□	□	□	□	□	□	□	0				
38,000*	Long	■	■	□	□	□	□	■	■	■	850	1339.07	50.94	3.72	10,604,193
	Short	□	□	□	□	□	□	■	■	■	489.07				
46,000	Long	■	■	■	□	□	□	■	■	■	965	1394.19	4.045	11.95	21,358,848
	Short	□	□	□	□	□	□	■	■	■	429.19				

*Base case

From table 4, for the first unit price, 30,000 baht, there are two long response LT facilities on location A and C, the production is 399 units short for satisfying demand and the profit is comparatively small at 1,454,690 baht. Hence, as the unit price decrease from 38,000 (base case) to 30,000 baht, it decides to opened just two long LT response facility to produce 1,000 units. In addition, it chooses to be short in production because in order to invest in opening one more facility, it may eventually turn the situation from gaining small amount of profit into loss. In particular, selling 399 units would not overcome the cost that they have to take to produce them.

For the second unit price, 46,000 baht, the profit is relatively increases as the selling price increases. According to table 4, as the unit price is higher, the long response facility in location A decide to add more capacity by opening the 3rd segment to sell more product for logical profit. The additional sale technically overcomes the cost of adding production capacity for all of the facility. Hence, the according unit price increases the production (comparing with the base case) from 1,339.07 to 1,394.19 and earns profit of 21,358,848 baht.

Facility fixed cost

Table 5:
Results of facility fixed cost variation

Type	***Facility fixed cost***	*Location*									*Production amount*		*Avg. Shortage amount*	*Avg. Overage amount*	*Total Profit*
		A			*B*			*C*							
		Segmentation													
		1	*2*	*3*	*1*	*2*	*3*	*1*	*2*	*3*	*Individual*	*Total*			
Long	3M	■	■	□	□	□	□	□	□	□	1050	1377.97	26.645	18.32	13,808,329
Short	5M	■	■	□	■	■	□	■	■	□	327.97				
Long	5M*	■	■	□	□	□	□	■	■	■	850	1339.07	50.94	3.72	10,604,192
Short	7M*	□	□	□	□	□	□	■	■	■	489.07				
Long	6M	■	■	□	□	□	□	■	■	■	850	1339.07	50.94	3.72	7,893,976
Short	8M	□	□	□	□	□	□	■	■	■	489.07				

*Base case

According to table 5, first, for [3M, 5M] facility fixed cost, we can obviously see the inverse relationship between facility fixed cost and total profit, as the fixed cost decrease from [5M, 7M], the profit increase from 10,604,192 to 13,808,329 baht. More importantly, this variation has highlighted the significance part of the decision. As the facility cost decrease, more facilities with smaller capacity are opened comparing with the base case. More importantly, all of the short response facilities are open even that they are more expensive. This case has proven that even the short LT facility has more fixed cost but it could overcome the cost to effectively handle the demand uncertainty.

Second, for [6M, 8M] facility fixed cost, as the fixed cost raises for 1 million baht for both type of facility, total profit drop from 10,604,192 to 7,893,976 baht. In this case, all of the decisions are the same as the base case, just the difference between the facility costs that cause the decline in profit.

Capacity additional cost

Table 6:
Results of capacity additional cost variation

<table>
<tr><th rowspan="4">Type</th><th rowspan="4">Capacity addition al cost</th><th colspan="9">Location</th><th colspan="2">Production amount</th><th rowspan="4">Avg. Shortage amount</th><th rowspan="4">Avg. Overage amount</th><th rowspan="4">Total Profit</th></tr>
<tr><th colspan="3">A</th><th colspan="3">B</th><th colspan="3">C</th><th rowspan="3">Individual</th><th rowspan="3">Total</th></tr>
<tr><th colspan="9">Segmentation</th></tr>
<tr><th>1</th><th>2</th><th>3</th><th>1</th><th>2</th><th>3</th><th>1</th><th>2</th><th>3</th></tr>
<tr><td>Long</td><td>500K</td><td>■</td><td>■</td><td>■</td><td>□</td><td>□</td><td>□</td><td>■</td><td>■</td><td>■</td><td>957</td><td rowspan="2">1391.77</td><td rowspan="2">5.335</td><td rowspan="2">10.815</td><td rowspan="2">13,304,165</td></tr>
<tr><td>Short</td><td>500K</td><td>□</td><td>□</td><td>□</td><td>□</td><td>□</td><td>□</td><td>■</td><td>■</td><td>■</td><td>434.77</td></tr>
<tr><td>Long</td><td>1M*</td><td>■</td><td>■</td><td>□</td><td>□</td><td>□</td><td>□</td><td>■</td><td>■</td><td>■</td><td>850</td><td rowspan="2">1339.07</td><td rowspan="2">50.94</td><td rowspan="2">3.72</td><td rowspan="2">10,604,193</td></tr>
<tr><td>Short</td><td>1M*</td><td>□</td><td>□</td><td>□</td><td>□</td><td>□</td><td>□</td><td>■</td><td>■</td><td>■</td><td>489.07</td></tr>
<tr><td>Long</td><td>2.5M</td><td>■</td><td>■</td><td>□</td><td>■</td><td>■</td><td>□</td><td>■</td><td>■</td><td>□</td><td>1050</td><td rowspan="2">1050</td><td rowspan="2">354.45</td><td rowspan="2">18.16</td><td rowspan="2">3,906,800</td></tr>
<tr><td>Short</td><td>2.5M</td><td>□</td><td>□</td><td>□</td><td>□</td><td>□</td><td>□</td><td>□</td><td>□</td><td>□</td><td>0</td></tr>
</table>

*Base case

From table 6, first, for 500K capacity additional cost, one significant thing that has changed from the base case is that the long LT response facility of the 500K cost has opened the 3rd segment as the cost of expansion is halved. According from the total production amount, as the 3rd segment is opened, the production from the less cost side is slightly higher than the 1M cost with 50 units. As a matter of fact, opening the 3rd segment reinforces 150 units more production capacity, but in this case, only about 50 units from potential of 150 units are produced which is surprisingly unexpected. Hence, the model made this decision because selling those 50 more units has overcome the cost of additional segment even that the leftover capacity is considerably abandoned.

Second, for 2M capacity additional cost, as the cost raises for 1 million baht, short LT response facility which has higher fixed cost are ignored to compensate the raise in capacity additional cost. From the base case to certain case, the short LT response facility in location C is closed and decides to open one more long LT response facility at location B to persuade the level of production. In addition, only at location A could afford to open three segments, the other two locations decided to open just 2 segments and result in 1,200 units in production which is about 100 units short. Therefore, as the capacity additional cost enhanced from 1M to 2M, the total profit reduces from 10,604,193 to 6,329,765 baht

Demand distribution

Table 7:
Results of demand variation

Demand Distribution		Type	Location ▯ A			B			C			Production amount		Avg. Shortage amount	Avg. Overage amount	Total Profit
Mean	SD		Segmentation 1	2	3	1	2	3	1	2	3	Individual	Total			
70	5	Long	■	■	■	■	■	□	■	■	■	1350	1350	64.14	23.49	11,945,595
		Short	□	□	□	□	□	□	□	□	□	0				
70	10	Long	■	■	□	□	□	□	■	■	■	850	1346.4	46.7	0	10,908,190
		Short	□	□	□	□	□	□	■	■	■	496.38				
70	15	Long	■	■	□	□	□	□	■	■	■	850	1338.1	52.26	3.9	10,613,078
		Short	□	□	□	□	□	□	■	■	■	488.14				
60	5	Long	■	■	□	□	□	□	■	■	□	1000	1000	191.3	0.92	9,868,849
		Short	□	□	□	□	□	□	□	□	□	0				
60	10	Long	■	■	□	□	□	□	■	■	□	1000	1000	207.8	17.33	9,146,386
		Short	□	□	□	□	□	□	□	□	□	0				
60	15	Long	■	■	□	□	□	□	■	■	□	700	1170.5	23.73	6.09	8,254,120
		Short	□	□	□	□	□	□	■	■	■	470.5				

*Base case

From table 7, first thing that is needed to be explain is that, it is practical that as the demand decline from mean of 70 units to 60 units, the total profit is also reasonably decline as well. The less of incoming demand, the less of sales it would be.

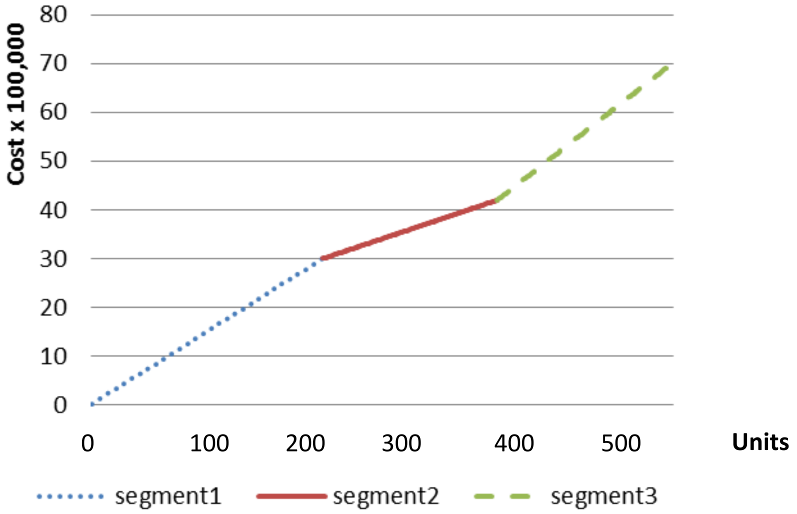
Second, by focusing on each mean of demand, for the mean of 70 units, we can obviously see that as the standard deviation (S.D.) raise from 5 to 10 then to 15, total profit is decreased from 11,945,595 to 10,908,190 then to 10,613,078 baht respectively. As well as for the mean of 60 units, as the S.D. raise from 5 to 10 then to 15, total profits are decreased from 9,868,849 to 9,146,386 then to 8,254,120 respectively. Hence, we can say that the less standard deviation of demand, the more profit could be performed.

Third, as we have proposed that short LT response facility helps handling the uncertainty of demand. The more standard deviation represents more degree of demand fluctuation or the level of uncertainty. In this part, for the mean of 70 units, as the S.D. reduce from 10 (base case) to 5, the short LT response facility is eliminated due to the cutback in level of uncertainty. In the other hand, for the mean of 60 units, as the S.D. enhance to 15, the short LT response facility is opened at maximum potential with 3 segments. Moreover, it pulls the production from the long LT response facility to itself to minimize the error from the fluctuation more effectively.

Cost function testing

The test is created based on 200 observations (scenarios) and base case's input data. The costs are representing in form of cumulative function. The slope indicated the cost in each segments. If the slope of the line is high (steep), the cost is more expensive, and vice versa. More importantly, the first segment has the capacity of 200 unit and 150 units each for the next 2 segments, so, each facility has the maximum possible capacity at 500 units. All of the 4 functions have weight average cost of 14,000 baht.

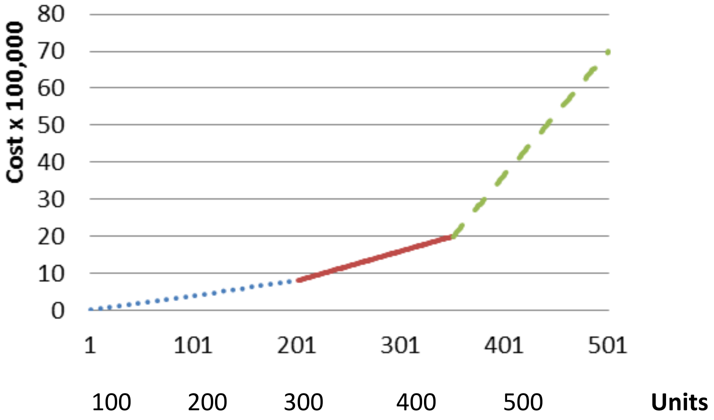
1.1.1 S-Shaped cost function



Cost: 15,000 8,000 18,666.67

Figure 2: S-shaped cost function

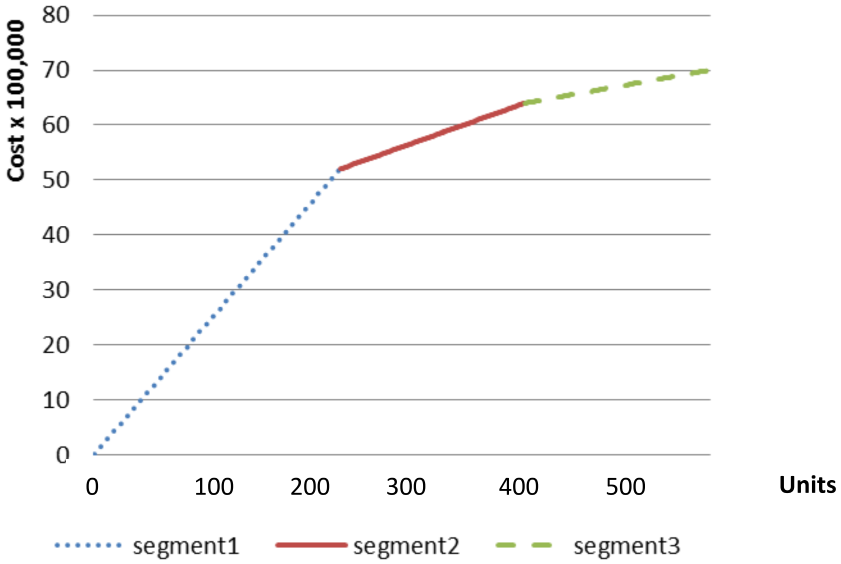
1.1.2 Convex cost function



Cost: 4,000 8,000 33,333.33

Figure 3: Convex cost function

1.1.3 Concave cost function



Cost: 26,000 8,000 4,000

Figure 4: Concave cost function

1.1.4 Stable cost function

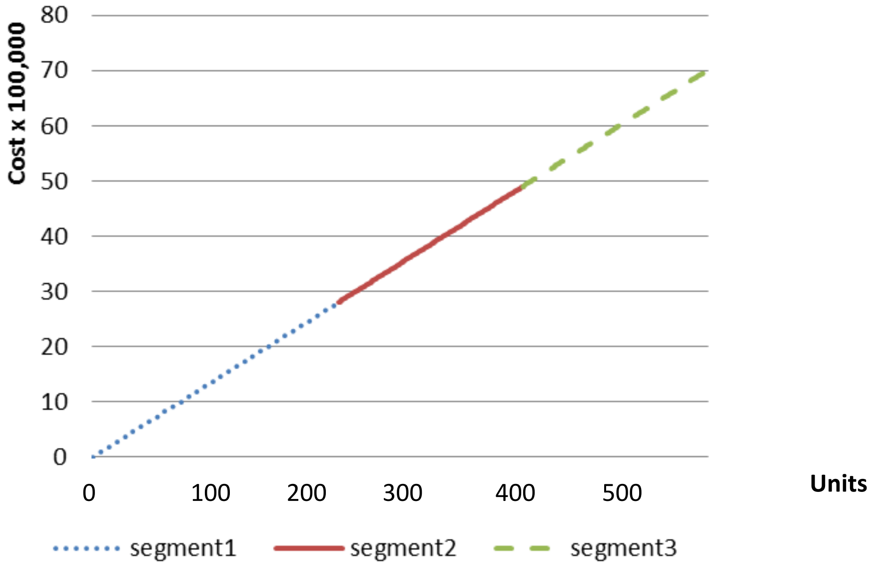


Table 8:
Results of 4 different cost function

Cost fn.	Type	Location ⍰									Production amount		Avg. Shortage amount	Avg. Overage amount	Total Profit
		A			B			C							
		Segmentation													
		1	2	3	1	2	3	1	2	3	Individual	Total			
S-shaped	Long	■	■	□	□	□	□	■	■	■	850	1346.16	41.275	0	10,903,814
	Short	□	□	□	□	□	□	■	■	■	496.16				
Convex	Long	■	■	□	■	■	□	■	■	□	1050	1379.15	11.575	3.29	15,552,714
	Short	□	□	□	□	□	□	■	■	□	329.15				
Concave	Long	■	■	■	□	□	□	■	■	■	1000	1000	386.42	1.86	9,827,047
	Short	□	□	□	□	□	□	□	□	□	0				
Stable	Long	■	■	□	□	□	□	■	■	■	850	1345.09	19.735	0	10,160,420
	Short	□	□	□	□	□	□	■	■	■	495.09				

From table 8, first, for the s-shaped cost function (figure 2), two segments for location A and three segments for location C of long LT response facility are opened. For short LT response, only facility from location C is opened with three segments. So in this case, location C is literally the critical location for minimizing the transportation cost in this framework as both long and short response facility are both opened at the same location. Besides, there are 850 units for long LT response facility and 496.16 units for short LT response which results in 1,346.15 units in total. Consequently, this cost function produces 10,903,814 baht for the profit.

Second, convex cost function (figure 3) achieves 15,552,714 baht profit which is comparatively higher than the s-shaped cost function. The first point to be focus is the nature of the function which the first 350 units from the first and second segment hold the cheap production cost and formidably increase for the last segment. Under those circumstances, from table 8, we obviously detect that the third segment are ignored in every opened facility. The results were decided to neglect the expensive cost segment and wisely chose to open one more facility at location B instead to keep up the level of production to serve demand. Ultimately, this function particularly makes more remarkable profit because as only first two segments are determined on each facility, the average cost has decently drop and overcome the cost of opening additional long LT response facility at location B.

Third, concave cost function (figure 4) announces the profit of 9,827,047 baht which is somewhat less than the s-shaped function. We observe that the facilities are opened at location A and C; both of them are all opened with 3 segments for 500 units maximum. According to the property of the cost function, the cost for first segment is very high comparing with the upcoming segments. The function itself undertakes the economies of scale role, as the higher quantity to be produced, the lower the average cost it is. In practical, economies of scale is the advantage for the supply chain member which they can acquire less average cost per unit as the inverse relationship between quantity and cost occur. Nevertheless, this situation arises differently, as total of 1,000 units are produced in long response facility, but it is still about 386.4 units short according to the demand. We accept that in this case, if any facility is opening, they all have to be fully capacitated to ensure the reasonable average cost. In the other hand, if one more facility is fully opened to satisfy the left over demand, it will create lots of overage unit and their penalty. Moreover, if one more facility is opening for one or two segments, the cost of production would no longer be suitable and could not overcome the cost of opening the additional facility.

Lastly, the stable cost function (figure 5), the result is apparently close to S-shaped cost function in many ways, the location and segmentation is the same, production and shortage are slightly different and has 743,394 baht less profit, so we could announce that s-shaped function is the most similar to linear average cost function which are casually implement in practical.

Conclusion

This paper proposed a stochastic programming for two stages supply chain with single-commodity, multi-facilities, multi-retailers, short response facility and piecewise cost function. We have developed 4 experiments to highlight the contributions of the model; (1) Sampling test, (2) Parameter variation, (3) Cost function testing and (4) Pair-T test.

First, for the sampling test, it literally shows that the standard deviation particularly lessens as the sample size amplified. So, we have proved that bigger sample size could produce more particular and precise results to mitigate the effect of uncertainty.

Second, for the parameter variation test, each parameter has there sensitiveness depend on their role in the model. Some parameter has significance effect on not just quantitative results but also qualitative results. One of the highlights in this part is the effect of standard deviation of the demand, we have proof that with lower S.D. results in higher demand fluctuation rate, the company can perform better and less contribution of short LT response facility is engaged. In the other hand, with higher standard deviation results in less degree of demand fluctuation, it is more difficult to control and relatively execute less profit and short LT response facility are more enforced in the system to handle the uncertainty.

Third, for the cost function testing, difference of cost function carries different nature which extremely impact on the facility segmentation and production capacity. S-shaped cost function and stable cost function seem to be familiar in term of profit and allocation. Next, the convex cost function is suitable with more site but smaller capacity facilities as the cost function itself incorporate the higher cost in the 3rd segment. Then, for the concave cost function, this function tends to be the most difficult function to compromise. With the higher cost in the first segment, all of the segments are needed to be utilized in order to acquire the suitable average cost for production in any opening facility.

And lastly, we can ultimately conclude that by applying long and short LT response together strategy is significantly better than using whether long LT response or short LT response only. Moreover, for the short LT facility, it has to be wisely and well-planned utilized as it requires extraordinary facility which is more potentially affordable.

Acknowledgement

This research was made possible by The BangchakPetroluem PCL.I gratefully acknowledge the funding received towards my master degree tuition scholarship at Sirindhorn International Institute of Technology (SIIT), Thammasat University under “The Bangchak Graduate Scholarship” program.

References

Amin, S.H. and Zhang, G.Q. (2013),“A Multi-Objective Facility Location Model for Closed Loop Supply Chain Net-work under Uncertain Demand and Return”,*Applied Mathematical Modeling*, Vol.37 Issue 6, pp. 4165-4176.

Chan, L.M.A., Muriel, A., Shen, Z.J.M. and Simchi-Levi, D. and Teo, C.P. (2002), “Effective Zero-Inventory-Ordering Policies for the Single-Warehouse Multiretailer Problem with Piecewise Linear Cost Structures”, *Management Science*, Vol.48 issue 11, pp. 1446-1460.

Chouinard, M.,D’Amours, S. andAït-Kadi, D. (2008),“A stochastic programming approach for designing supply loops”, *International Journal of Production Economics*, Vol.113 Issue 2, pp. 657-677.

Diabat, A. and Theodorou, E. (2015), “A Location-Inventory Supply Chain Problem: Reformulation and Piecewise Linearization”, available at
doi: http://dx.doi.org/10.1016/j.cie.2015.05.021.

Goh, M.,Lim and J.Y.S,Meng, F. (2007),“A stochastic model for risk management in global supply chain networks”, *European Journal of Operational Research*, Vol.182 Issue 1, pp. 164-173.

Hendricks, K. B. andSinghal, V. R. (2005), “Association between supply chain glitches and operating performance”, *Management Science.*, Vol.51 No. 5, pp. 695–711.

Jouzdani, J.,Sadjadi, S.J. andFathian, M. (2013),“Dynamic dairy facility location and supply chain planning under traffic congestion and demand uncertainty: A case study of Tehran”, *Applied Mathematical Modelling*, Vol.37 Issues 18–19, pp. 8467-8483,

Kaki, A., Salo, A. andTalluri, S. (2014), “Scenario-Based Modeling of Interdependent Demand and Supply Uncertainties”, *Engineering Management, IEEE Transactions on*, Vol.61 No.1, pp. 101,113.

Lee, H., Padamanabhan, P. and Whang, S. (1997), “Information distortion in supply chain: the bullwhip effect”, *Management Science*, Vol.43 No.4, pp. 546–558.

Melo, M.T., Nickel, S. andSaldanha-da-Gama F. (2006), “Dynamic multi-commodity capacitated facility location: a mathematical modeling framework for strategic supply chain planning”, *Computers & Operations Research*, Vol.33 No. 1, pp. 181–208.

Thanh, P.N., Bostel, N. and Péton, O. (2008), “A dynamic model for facility location in the design of complex supply chains”, *International Journal of Production Economics*, Vol.113 No.2, pp. 678–693.

Wang, K.J.,Makond, B. and Liu, S.Y. (2011),“Location and allocation decisions in a two-echelon supply chain with stochastic demand – A genetic-algorithm based solution”, *Expert Systems with Applications*, Vol. 38 Issue 5, pp. 6125-6131.

Zhang, W. and Xu, D. (2014), “Integrating the logistics network design with order quantity determination under uncertain customer demands”, *Expert Systems with Applications*, Vol.41 Issue 1, pp. 168-175.